1. Sabiendo que
$$\mathcal{L}\{e^t f(t)\}(s) = \frac{1}{s^2 - 2s + 2}.$$
Calcular:
$$\mathcal{L}\left\{e^{3t}\frac{f(t)}{t}\right\}(s), \quad \mathcal{L}\{tf(t)\}(s) \quad \text{y} \quad \int_0^\infty \frac{f(t)}{t}\,dt.$$

2. Demostrar que
$$\int_0^\infty e^{-st}\frac{1-\cos(t)}{t^2}dt = \frac{\pi}{2} + \frac{s}{2}\log\left(\frac{s^2}{s^2+1}\right) - \arctan(s).$$

3. Demostrar que
$$\mathcal{L}\left\{\frac{\operatorname{sen}^2(t)}{t}\right\} = \frac{1}{4}\log\left(\frac{s^2+4}{s^2}\right).$$

4. Sabiendo que
$$\mathcal{L}\{e^t f(t)\}(s) = \frac{s+1}{s^2 - 2s + 1}.$$
Se pide $\mathcal{L}\left\{\frac{f(3t)}{t}\right\}(s)$ y $f(3t)$.

5. Sabiendo que
$$\mathcal{L}\{e^t f(t)\} = \log\left(\frac{s+1}{s-1}\right).$$
Hallar $\mathcal{L}\{tf(2t)\}$ y $f(2t)$.

6. Calcular $\mathcal{L}\{e^{-t}f(2t)\}$ sabiendo que
$$\mathcal{L}\{tf(t)\} = \frac{1}{s(s^2+1)}.$$
Calcular también $f(t)$.

7. Calcular
$$\mathcal{L}^{-1}\{\arctan(1/s)\}, \quad \text{y} \quad \mathcal{L}\left\{\frac{\operatorname{sen}(t)\cos(t)}{t}\right\}.$$

8. Probar que
$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{2a^3}\operatorname{sen}(at) - \frac{1}{2a^2}t\cos(at).$$

9. Sabiendo que $\mathcal{L}\{\operatorname{sen}(at)\} = \frac{a}{s^2+a^2}$, demostrar que
$$\int_0^\infty \frac{e^{-\sqrt{2}t}\operatorname{senh}(t)\operatorname{sen}(t)}{t}dt = \frac{\pi}{8}$$
Recordar que
$$\arctan\left(\frac{A-B}{1+AB}\right) = \arctan(A) - \arctan(B).$$

10. Utilizando la convolución, hallar

$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^4+s^2}\right\}.$$

11. Calcular $\mathcal{L}\{f(t)\}(s)$, siendo

$$f(t)=\frac{\cos(at)-\cos(bt)}{t}.$$

12. Calcular $\mathcal{L}^{-1}\left\{\log\left(\frac{2s-1}{2s+1}\right)\right\}$

13. Sabiendo que

$$\mathcal{L}\{e^{-t}f(t)\}(s)=\log\left(\frac{s-1}{s+1}\right).$$

Calcular $\mathcal{L}\{f(t)\}(s)$, $\mathcal{L}\{tf(2t)\}(s)$, y $f(2t)$.

14. Sabiendo que

$$\mathcal{L}\{e^{2t}f(t)\}(s)=\log\left(\frac{s+2}{s-2}\right).$$

Calcular $\mathcal{L}\{f(t)\}(s)$, $\mathcal{L}\{tf(t/3)\}(s)$, y $f(t/3)$.