Practice exercises

Matrices and Linear Systems. Gauss-Jordan method – Survey

1. Apply the factorization LU decomposition to matrix A to solve the linear system $Ax = b$, where
$$\begin{pmatrix} 1 & 2 \\ 4 & 9 \end{pmatrix}$$
and $b = (5, 21)^T$.
Notation: Here X^T means the transpose of the matrix X.

2. Given the 2-by-2 symmetric matrix
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$
obtain its LDL^T factorization, where D is a diagonal matrix and L is unit lower triangular.

3. Given the 3-by-3 symmetric matrix
$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$
obtain its LDL^T factorization.

4. Given the 3-by-3 symmetric matrix
$$A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 7 & 5 \\ -2 & 5 & 30 \end{pmatrix}$$
obtain its LDL^T factorization.

5. Let A be the 4-by-4 Pascal Matrix:
$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}$$
Obtain its LU decomposition and use such decomposition to solve the linear equation $Ax = b$ where $b = (8, 20, 40, 70)^T$.

6. Let A be the 4-by-4 tridiagonal matrix:

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 4 \end{pmatrix}$$

Obtain its LU decomposition and use such decomposition to solve the linear equation $Ax = b$ where $b = (3, 6, 6, 7)^T$.

7. Let A be the symmetric matrix

$$A = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 6 \end{pmatrix}$$

Obtain the LDL^T decomposition of A and of its opposite $-A$.

From the results we deduce that A is positive definite and $-A$ is negative definite.

8. Let A be the symmetric matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

(a) Obtain its LDL^T decomposition.
(b) Compute the inverse of A , if it exists, by using the Gauss-Jordan method.

9. Let us consider the matrices

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

Obtain, by using the Gauss-Jordan method, the inverse of A, B and AB. Check that $(AB)^{-1} = B^{-1}A^{-1}$.

10. Let A be the 2-by-2 matrix:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

By using the Gauss-Jordan method, compute the inverse matrix of A and of its transpose $A^T$and check that $(A^T)^{-1} = (A^{-1})^T$.

11. Let A be the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 6 & 6 \\ 2 & 11 & 13 \end{pmatrix}$$

(a) Obtain its LU decomposition.

(b) Solve the linear system $Ax = (1, 3, 0)^T$ by using the decomposition obtained in (a).

12. Let B the matrix

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 6 & 6 \\ 2 & 11 & k \end{pmatrix}$$

(a) Obtain the values for k so that B is singular.

(b) Find the values for k so that the linear system $Bx = (2, 9, 13)^T$ has infinitely many solutions. Obtain the general solution for such a case.

(c) Find the values for k so that the linear system $Bx = (2, 9, 13)^T$ has one solution. Obtain such solution.

13. Let

$$A = \begin{pmatrix} 1 & 1 & 2 & 0 & 0 \\ 1 & 2 & 2 & 2 & 4 \\ -1 & 0 & -2 & 2 & 4 \end{pmatrix}$$

(a) Solve the homogeneous system $Ax = 0$.

(b) Solve the underdetermined linear system $Ax = (2, 4, 0)^T$.

(c) Given three values, namely b_1, b_2, and b_3. Find the linear system such values need to fulfill so that the linear system $Ax = (b_1, b_2, b_3)^T$ has solution.