Practice exercises

Diagonalization – Survey

1. Study if the following matrices can be diagonalized over $\mathbb{R}$ and, if so, obtain matrices D (Diagonal) and P (regular) so that $A = PDP^{-1}$.

$$(a)\, A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \quad (b)\, A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad (c)\, A = \begin{pmatrix} 1 & 0 & -1 & 1 \\ -2 & -1 & 2 & -4 \\ -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 3 \end{pmatrix}.$$

2. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear mapping which has the following matritial representation with respect to the canonical basis:

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & a & 0 \\ 2 & 1 & -1 \end{pmatrix}.$$

   (a) Find the values of a so that A is diagonalizable over $\mathbb{R}$.

   (b) For $a = 3$ obtain the matrices D (diagonal) and P (regular) such that $A = PDP^{-1}$.

3. Let us consider the matrix

$$A = \begin{pmatrix} b & 0 & -2b \\ -1 & 1 & 2 \\ -b & 0 & 2b \end{pmatrix}$$

   that depends on the real parameter b.

   a) Determine the values of b so that A is diagonalizable over $\mathbb{R}$.

   b) For $b = 0$ find the matrices D (diagonal) and P (regular) so that $A = PDP^{-1}$.

   c) Compute, As application of (b), the power A^n (for the $b = 0$ case) for any positive integer n.

4. Let $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear mapping defined by:

$$f(x, y, z) = (\alpha x - \alpha y + 3z, -2\alpha x + 2\alpha y + z, 3z).$$

   where α is real.

   (a) Compute A the coordinate matrix with respect to the canonical basis.

(b) Determine the values of α so that A is diagonalizable over $\mathbb{R}$.

(c) For $\alpha = -1$ find the matrices D (diagonal) and P (regular) so that $A = PDP^{-1}$.

(d) Compute, as application of the precious result, the power A^n (fot the $\alpha = -1$ case) as a function on n. Simplify as mush as possible.

5. Let us consider the matrix

$$A = \begin{pmatrix} b & 2 & 0 \\ 2 & b & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

that depends on the real parameter b.

a) Study the values of b for whose the matrix A is diagonalizable over $\mathbb{R}$

b) For the $b = 1$ case obtain matrices D (diagonal) and P (regular) so that $A = PDP^{-1}$.

c) Obtain, as an application of the previous result, the power A^n (in the $b = 1$ case) for any positive integer as a function on b. Simplify it as much as possible.

6. Let us consider the matrix

$$A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

that depends on the real parameter a.

a) Study the values of a for whose the matrix A is diagonalizable over $\mathbb{R}$

b) For the $a = 1$ case obtain matrices D (diagonal) and P (regular) so that $A = PDP^{-1}$.

c) Obtain, as an application of the previous result, the power A^n (in the $a = 1$ case) for any positive integer as a function on b. Simplify it as much as possible.

7. Let us consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 0 & -2 & a \end{pmatrix}$$

that depends on the real parameter a.

a) Study the values of a for whose the matrix A is diagonalizable over $\mathbb{R}$

b) For the $a = 3$ case obtain matrices D (diagonal) and P (regular) so that $A = PDP^{-1}$.

c) Obtain, as an application of the previous result, the power A^n (in the $a = 3$ case) for any positive integer as a function on b. Simplify it as much as possible.

8. Let us consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 2 & 0 & b \end{pmatrix}$$

that depends on the real parameter b.

a) Study the values of b for whose the matrix A is diagonalizable over $\mathbb{R}$

b) For the $b = 2$ case obtain matrices D (diagonal) and P (regular) so that $A = PDP^{-1}$.

c) Obtain, as an application of the previous result, the power A^n (in the $b = 2$ case) for any positive integer as a function on b. Simplify it as much as possible.