Practice exercises

Linear Differential Equations – Survey

1. Solve the following initial values problem:

$$\begin{cases} u' &= Au \\ u(0) &= (1,1,1)^T \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

2. Solve the following initial values problem where the initial value is given at $t = 1$:

$$\begin{cases} u' &= Au \\ u(1) &= (8e^4 - 5e, 5e)^T \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix}.$$

3, Solve the following initial values problem:

$$y''(t) - y'(t) - 2y(t) = e^t, \qquad y(0) = 1,\ y'(0) = 3.$$

4. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) + B(t) \\ u(0) &= (1,1)^T \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix},\ B(t) = (3e^{2t}, 4e^{2t})^T.$$

5. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) \\ u(0) &= (1,1)^T \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}.$$

6. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) \\ u(0) &= (500, 100)^T \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}.$$

7. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) \\ u(0) &= (-1,-2,-30)^T, \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{pmatrix}$$

8. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) \\ u(0) &= (1,0,0)^T, \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}.$$

9. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) \\ u(0) &= (2,2)^T, \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

10. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) \\ u(0) &= (1,2)^T, \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix}.$$

11. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) + B(t) \\ u(0) &= (1,1)^T, \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix},\ B(t) = (2,3)^T.$$

12. Solve the following initial values problem:

$$\begin{cases} u'(t) &= Au(t) + B(t) \\ u(0) &= (7,9)^T, \end{cases} \qquad \text{where} \qquad A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},\ B(t) = (0,t)^T.$$

12. Solve the following initial values problem:

$$\begin{cases} y'''(t) - 2y''(t) - y'(t) + 2y(t) = 0 \\ \quad y(0) = 3,\ y'(0) = 2,\ y''(0) = 6 \end{cases}$$