

Problem Set 7

Boolean Algebra – Survey

1. Simplify the following Boolean expressions:

(a) $x + y + \overline{(\bar{x} + y + z)}$

(b) $(\bar{x} + yz(x + y))x + (z + \bar{x})(z + y + \bar{w})$

(c) $x_1 + \bar{x}_1x_2 + \bar{x}_1\bar{x}_2x_3 + \bar{x}_1\bar{x}_2\bar{x}_3x_4$

2. Write the tables corresponding to the Boolean functions

(a) $f(x, y, z) = xy + \bar{z}$

(b) $g(x, y, z) = x\bar{y} + yz.$

3. Determine the expressions of the Boolean functions f and g whose tables are given:

x	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1
$f(x, y, z)$	0	1	0	1	0	1	0	1
$g(x, y, z)$	1	1	1	1	0	0	1	1

4. Determine the disjunctive normal form (d.n.f.) and the conjunctive normal form (c.n.f.) of the functions

(a) $f(x, y, z) = (x + y)(x + z)$

(b) $g(x, y, z) = x(y + \bar{z}).$

5. Determine the conjunctive normal form of the functions:

(a) $f(\omega, x, y, z) = (\omega + x)(\omega + \bar{y} + z)(x + y + \bar{z})$

(b) $g(\omega, x, y, z) = \bar{z}(x + \omega) + (x + \omega)\bar{y}.$

6. Given $f(\omega, x, y, z) = \sum m(5, 7, 10, 13, 14, 15)$, determine its conjunctive normal form.

7. Find the d.n.f. and the c.n.f. of $f(x, y, z) = \overline{(x + y) + x\bar{z}}$.

8. Determine if the following Boolean functions are equal or not:

$$f(x, y, z) = \bar{x}y + \bar{y}z + x\bar{y}, \quad \text{and} \quad g(x, y, z) = x\bar{y} + y\bar{x} + z\bar{x} + z\bar{y}.$$

9. Given $f(\omega, x, y, z) = \sum m(4, 5, 7, 8, 9, 11)$, find its **d.n.f.** and its **c.n.f.** and a simplified minimal sum.
10. Given the Boolean functions $f(\omega, x, y, z) = \sum m(0, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 15)$ and $g(\omega, x, y, z) = \sum m(3, 4, 6, 7, 9, 11, 12, 13, 14, 15)$, we want to:
- Find the tables of f , g , \bar{f} , $f + g$, and $f \cdot g$.
 - Find the **d.n.f.** and the **c.n.f.** of f and g .
 - Construct a circuit (with logic gates) for f , using its **c.n.f.** and its **d.n.f.**.
 - Determine a simplified minimal sum for f and g .
 - Check if the Boolean functions $h_1(\omega, x, y, z) = \bar{w}\bar{z} + \omega\bar{y} + yz$ and $h_2(\omega, x, y, z) = \bar{w}y + y\bar{z} + \omega z$ are equal or not. Does any of them coincide with f ?
11. Let f and g be the Boolean functions which are defined by the following table:

ω	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
x	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$f(x, y, z)$	0	1	1	1	1	1	0	0	0	1	1	1	1	1	0	0
$g(x, y, z)$	0	0	0	1	0	1	0	0	1	0	1	0	1	1	1	1

- Write f and g as a product of maxterms.
 - Simplify f and g using the following Karnaugh's map, explicitly mentioning the minterms corresponding to each block, and their simplification.
12. Use the Karnaugh's method for simplifying the Boolean function??

$$f(\omega, x, y, z) = m(0, 1, 3, 4, 7, 8, 11, 12, 13, 15).$$