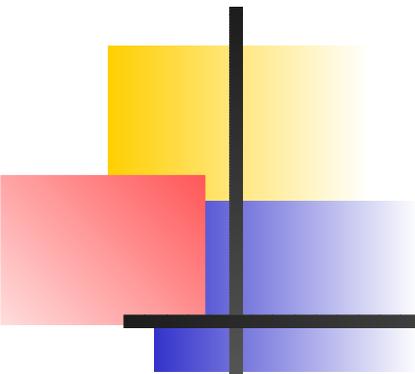


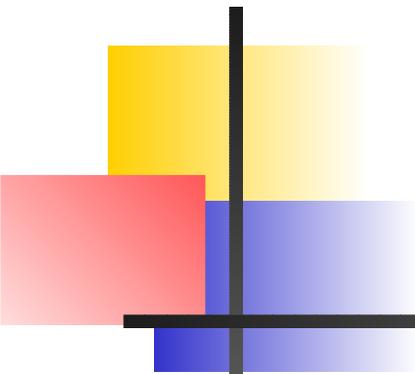
La función zeta de Riemann y sus q-extensiones. Aplicaciones

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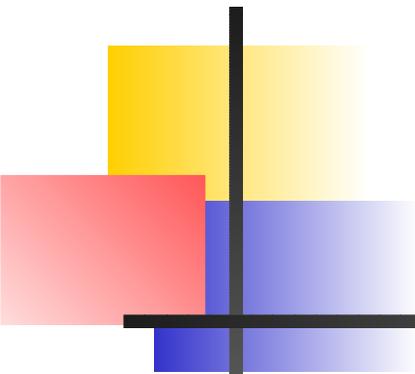
0. Referencias

- The theory of Riemann Zeta function.
[Titchmarsh](#) (1951)
- Concrete Mathematics.
Graham, Knuth, Patashnick (1994)
- Trabajos de Apéry, Borwein, Erdős, Gel'fond,
W. Van Assche, etc
- D.E.A. (2005)



1. Motivación

- Propiedades algebraicas de Zeta Riemann (1859)(L-funciones),
- Zeros no triviales (hipótesis de Riemann, hipótesis de Lindelöf),
- Expresión analítica e irracionalidad de $\zeta(2k + 1)$,
- Independencia irracional.



2. Teoría de Números

- > L-funciones: Grado 1, 2 y clase S (Selberg)+Axiomas.
- > $\varphi(n)$, $\mu(n)$, $\Lambda(n)$ Mangoldt , ..
- > Función multiplicativa,
- > Principio de inversión.
- > Polinomios ciclotómicos. Propiedades
- > Teorema de los números primos: $\pi(x) \sim \frac{x}{\log x}$.

3. Algunas identidades

- $$\frac{1}{\zeta(s)} = \sum_{n \geq 1} \frac{\mu(n)}{n^s},$$
 - $$-\frac{\zeta'(s)}{\zeta(s)} = \sum_{n \geq 1} \frac{\Lambda(n)}{n^s},$$
 - $$(1 - 2^{s-1})\zeta(s) = \sum_{n \geq 1} \frac{(-1)^{n-1}}{n^s},$$
- $$\zeta(s) = \prod_{p \text{ primo}} (1 - p^{-s})^{-1}.$$

■ 4. q -extensiones

- Fixed $\beta, q \in \mathbb{C}$, $q^\beta \neq 1$, the q^β -number

$$[n; \beta]_q \equiv [n]_{q^\beta} = \frac{q^{n\beta} - 1}{q^\beta - 1}.$$

- $(\overline{\{[n; \beta]_q\}_{n \in \mathbb{Z}}}, +, *)$ is a Field, y un DFU.
- $[n; \beta]_q$ is q -prime iff exists an irreducible polynomial $f \in \mathbb{Z}[x] : [n; \beta]_q = f(q^\beta)$.

And is q -composite in other case.

4. $\zeta(\mathbb{Z})$. Identidades

- $\Gamma\left(\frac{s}{2}\right) \pi^{-\frac{s}{2}} \zeta(s) = \Gamma\left(\frac{1-s}{2}\right) \pi^{-\frac{1-s}{2}} \zeta(1-s).$

- $\zeta(2n).$

- **New Function:** Let n be a positive integer,

$$\zeta_{\Phi}(n) = \prod_{p \text{ prime}} [\Phi_1^{\mu(n)}(1/p) \Phi_n^{-1}(1/p)].$$

- $\zeta(n) = \prod_{t|n} \zeta_{\Phi}(t) \Rightarrow \prod_{1 \neq k|n} (\zeta(k))^{\mu(n/k)} = \zeta_{\Phi}(n).$

5. Hipótesis de Riemann

- (RH): $\zeta\left(\frac{1}{2} + it\right) = \mathcal{O}(|t|^\epsilon), \quad \epsilon \ll 1.$
- (RH) $\iff \sum_{n \leq x} \Lambda(n) = x + \mathcal{O}(\sqrt{x}), \quad x \gg 1.$
- $\sum_{n \leq N} \Lambda(n) = \log(\text{mcm}(1, 2, \dots, N)).$
- Definimos r_n, u_n y d_n .
- $d_{n+1} r_{n+1}^{\frac{1}{2}} - u_n (r_{n+1} - 1)^{\frac{1}{2}} = \log(p(r_{n+1})) - 1.$